
Definition: The solution set of a homogeneous linear system $A\mathbf{x} = \mathbf{0}$ is called the **general solution**. A constant vector \mathbf{x}_0 in the solution set of a consistent linear system $A\mathbf{x} = \mathbf{b}$ is a **particular solution**.

Theorem 4.8.2 If \mathbf{x}_0 is any solution of a consistent linear system $A\mathbf{x} = \mathbf{b}$, and if $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is a basis for the null space of A , then every solution of $A\mathbf{x} = \mathbf{b}$ can be expressed in the form $\mathbf{x} = \mathbf{x}_0 + c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \dots + c_k \mathbf{v}_k$. Conversely, for all choices of scalars c_1, c_2, \dots, c_k , the vector \mathbf{x} in this formula is a solution of $A\mathbf{x} = \mathbf{b}$.

Definition: For an $m \times n$ matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ the vectors

$$\begin{aligned} \mathbf{r}_1 &= [a_{11} \ a_{12} \ \cdots \ a_{1n}] \\ \mathbf{r}_2 &= [a_{21} \ a_{22} \ \cdots \ a_{2n}] \\ &\vdots \\ \mathbf{r}_m &= [a_{m1} \ a_{m2} \ \cdots \ a_{mn}] \end{aligned}$$

in R^n that are formed from the rows of A are called the **row vectors** of A , and the vectors

$$\mathbf{c}_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix}, \dots, \mathbf{c}_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

in R^m that are formed from the columns of A are called the **column vectors** of A .

Definition: If A is an $m \times n$ matrix, then the subspace of R^n spanned by the row vectors of A is denoted by $\text{row}(A)$ and is called the **row space** of A , and the subspace of R^m spanned by the column of vectors of A is denoted by $\text{col}(A)$ and is called the **column space** of A . The solution space of the homogeneous system of equations $A\mathbf{x} = \mathbf{0}$, which is a subspace of R^n , is denoted by $\text{null}(A)$ and is called the **null space** of A .

